

Course Code
Math 1
Module 1, Academic Year 2025-26

Course Information

Instructor:

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Office Hour: Mon & Thur 15:30-17:00; Wed: 10:30-12:00

Teaching Assistant: Danyi Shu

Phone:

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Classes:

Lectures: Mon & Thur 13:30-15:20

Venue: PHBS Building, Room 321

Course Website:

If any.

1. Course Description

1.1 Context

Course overview: This course provides PhD students in economics with a rigorous foundation in mathematical tools essential for modern economic theory and dynamic analysis. It begins with core topics in real analysis, covering the topology of Euclidean spaces, sequences and series, and conditions for convergence. Building on this, students are introduced to metric and normed spaces, with an emphasis on completeness, compactness, and continuity, leading to key theorems such as Banach's Contraction Mapping, Arzelà-Ascoli, and the Baire Category theorem. The course then develops measure theory, including σ -algebras, Lebesgue integration, convergence theorems, and L^p spaces, which form the basis for probability, expectations, and econometrics. With these analytical foundations in place, the course turns to applications in optimal control theory: first in deterministic settings, covering the Euler equations and Pontryagin's Maximum Principle, and then in stochastic environments, introducing Brownian motion, Itô calculus, and Hamilton-Jacobi-Bellman equations.

Prerequisites: Basic calculus and linear algebra

1.2 Textbooks and Reading Materials

Pugh's Real Mathematical Analysis

Acemoglu's Introduction to Modern Economic Growth (mathematical chapters)

Lecture notes

2. Learning Outcomes

2.1 Intended Learning Outcomes

Learning Goals	Objectives	Assessment (YES with details or NO)
1. Our graduates will be effective communicators.	1.1. Our students will produce quality business and research-oriented documents.	No
	1.2. Students are able to professionally present their ideas and also logically explain and defend their argument.	Yes
2. Our graduates will be skilled in team work and leadership.	2.1. Students will be able to lead and participate in group for projects, discussion, and presentation.	Yes
	2.2. Students will be able to apply leadership theories and related skills.	No
3. Our graduates will be trained in ethics.	3.1. In a case setting, students will use appropriate techniques to analyze business problems and identify the ethical aspects, provide a solution and defend it.	Yes
	3.2. Our students will practice ethics in the duration of the program.	No
4. Our graduates will have a global perspective.	4.1. Students will have an international exposure.	No
5. Our graduates will be skilled in problem-solving and critical thinking.	5.1. Our students will have a good understanding of fundamental theories in their fields.	Yes
	5.2. Our students will be prepared to face problems in various business settings and find solutions.	No
	5.3. Our students will demonstrate competency in critical thinking.	Yes

2.2 Course specific objectives

The objective of this course is to equip PhD students in economics with the analytical tools and proof techniques needed to read, produce, and evaluate modern theory under certainty and uncertainty. By the end, students should (i) master core real analysis on $\mathbb{R}^n$, metric spaces, and measure/integration; (ii) use Banach contraction, Arzelà–Ascoli, and Baire category to establish existence, uniqueness, continuity, and stability of equilibria and value functions; and (iii) formulate and solve deterministic and stochastic optimal control problems via Euler/Pontryagin methods, dynamic programming, Itô calculus, and HJB equations. Emphasis is on rigorous argumentation, correct use of convergence theorems, and translating mathematical results into economic insights and model validation.

2.3 Assessment/Grading Details

Assessment for the course will be based on a combination of assignments (20%), a midterm (40%), and a final exam (40%)

2.4 Academic Honesty and Plagiarism

It is important for a student's effort and credit to be recognized through class assessment. Credits earned for a student work due to efforts done by others are clearly unfair. Deliberate dishonesty is considered academic misconducts, which include plagiarism; cheating on assignments or examinations; engaging in unauthorized collaboration on academic work; taking, acquiring, or using test materials without faculty permission; submitting false or incomplete records of academic achievement; acting alone or in cooperation with another to

falsify records or to obtain dishonestly grades, honors, awards, or professional endorsement; or altering, forging, or misusing a University academic record; or fabricating or falsifying of data, research procedures, or data analysis.

All assessments are subject to academic misconduct check. Misconduct check may include reproducing the assessment, providing a copy to another member of faculty, and/or communicate a copy of this assignment to the PHBS Discipline Committee. A suspected plagiarized document/assignment submitted to a plagiarism checking service may be kept in its database for future reference purpose.

Where violation is suspected, penalties will be implemented. The penalties for academic misconduct may include: deduction of honour points, a mark of zero on the assessment, a fail grade for the whole course, and reference of the matter to the Peking University Registrar.

AI tools requirements:

Using AI tools to complete assignments or assessments without the approval of the course instructor will be regarded as an act of academic dishonesty. Depending on the severity of the situation, penalties will be implemented in accordance with the provisions of the Peking University Graduate Student Handbook.

For more information of plagiarism, please refer to *PHBS Student Handbook*.

3. Topics, Teaching and Assessment Schedule

Sept. 1, 4 — Introduction & review of real analysis: topology of $\mathbb{R}^n$, open/closed sets, compactness, connectedness.

Sept. 8, 11 — Sequences and series: convergence, uniform convergence, interchange of limits.

Sept. 15, 18 — Metric spaces: definitions, examples, completeness, compactness, continuity.

Sept. 22, 25 — Banach spaces (basics) and three core theorems: Banach Contraction Mapping, Arzelà–Ascoli, Baire Category.

Sept. 29, Oct. 9 — Applications of fixed-point theorems and existence/uniqueness of equilibria; review session.

Oct. 13 — Midterm Exam (covers real analysis, metric spaces, fixed-point theorems).

Oct. 16, 20 — Measure theory: sigma-algebras, Lebesgue measure, Lebesgue integration.

Oct. 23 — Convergence theorems (monotone, dominated convergence, Fatou's lemma).

Oct. 27 — L^p spaces: definitions, completeness, Hölder and Minkowski inequalities.

Oct. 30 — Applications of measure theory to probability, expectations, and econometrics.

Nov. 3 — Deterministic optimal control: Euler equations, Pontryagin Maximum Principle, applications (Ramsey model).

Nov. 6 — Stochastic optimal control: Brownian motion, Itô's lemma, HJB equations; course wrap-up and final review.

4. Miscellaneous